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ANALYSIS OF THE THREE LOWEST BENDING FREQUENCIES OF A ROTATING PROPELLER

By F. Liebers

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ANALYSIS OF THE THREE LOWEST BENDING FREQUENCIES OF A ROTATING PROPELLER*

By F. Liebers

SUMMARY

The available literature on rotating propeller oscillations reveals a lack of uniformity in interpretation, particularly as concerns the data on the overtone frequency with respect to the centrifugal forces.

The present report is a survey of the existing data for computing the bending frequency and a check on the dependability of the calculating methods.

INTRODUCTION

Of the possible propeller oscillation modes, only the bending oscillations have been explored to any considerable extent. There is no longer any doubt about the occurrence of the fundamental mode and the first and second overtones in bending. A number of adequate causes to excite the oscillation are also known. The occasionally voiced opinion that, owing to the great air damping of the overtones, the exciting forces caused by the engine are too small to incite flexural oscillations of dangerous amplitude, is hardly justified. A rough calculation for a practical example revealed that the amplitudes of the first overtone would have to reach amounts of the order of magnitude of ±5 cm (1.97 in.) at the free blade tip, to be capable of equalizing the exciting forces set up by the engine. All damping other than air damping was, of course, disregarded thereby, and allowed for in the customary semi-steady fashion.

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*"Zur Berechnung der 3 tiefsten Biegofrequenzen der umlaufenden Schraube." Luftfahrtforschung, August 31, 1935, pp. 155-160.

But for all other investigations, the exact knowledge of the oscillation frequencies of the rotating propeller is of fundamental importance. And even here there still seems to prevail a certain doubtfulness. For example, F. W. Caldwell, in a recent report, gives new coefficients again for the frequency formulas which are markedly unlike the figures known heretofore, particularly for the overtones. On the other hand, M. Hansen and G. Mesmer's report, published in 1933, which proved the occurrence of overtones in experiments, failed to give a correct picture of the previous experiments, which chiefly concerned the fundamental mode.

The chief aim in the following is to survey the existing data for computing the frequency of the rotating propeller with a view to elucidating the actually attained dependability factor, at least in this point. This affords, at the same time, a supplement to the calculation of the overtones. A definite knowledge of the harmonics at the second overtone frequency may, for example, become of importance when applied to the coupling with torsional oscillations for frequencies of the same order of magnitude.

Fundamental Mode

The frequency of a rotating propeller is usually expressed by

$$\lambda^2 = \lambda_0{}^2 + C\,\omega^2 \tag{1}$$

(ω = revolutions, C = constant = centrifugal force coefficient, $\lambda_0 = \lambda_{\omega=0}$ = static frequency). Various writers have computed a number of such formulas (reference 7) for specially simple bar shapes, wherein constants λ_0 and C in (1) assumed different values, depending on the particular visualization for the bar representing the propeller. As a result, the frequency formula established for an example considered typical, had to be considered as being of general validity for real propellers. But this made the problem subject to certain inaccuracies which, for the centrifugal force coefficient C, caused a scatter of 40 percent or more. To illustrate: For the two idealized propellers which had (1) a rectangular section with the moment of inertia varying with the cube of the length, and (2) a rectangular section with the moment of inertia varying with the square of the length - the centrifugal force coefficients, $C = 1.52$ and $C = 1.08$, were computed. If, on the other hand, such discrepancies were deemed too serious - and rightly so - then it became necessary to ascertain the elastic line from the exactly defined bar form of every single propeller, and then determine the C factor from it each time, as was necessary in Southwell and Gough's report (reference 2). The mathematical treatment of the centrifugal force effect is quite tedious, especially if it includes the overtones. For this reason, it has never been attempted except in ideal cases, such as cited above.

In point of fact, the conditions relative to the frequency rise of the rotating propeller is far more simple than the discrepancies in the data of the older literature seem to indicate. As they were all based on the elastic line of the stationary bar, they are, strictly speaking, applicable only for very low values of r.p.m. And for these, the factor C is subordinate, because the percentage of frequency increase itself is small. At the maximum number of revolutions $\frac{\omega}{\lambda_0} \approx 1.5$ in question for the propeller, the frequency rise due to centrifugal force amounts, however, to about 1,000 percent, and as much as 50 percent even in the practically more important $\frac{\omega}{\lambda_0} \approx 1$ r.p.m. range. For these conditions the leveling effect of the centrifugal force must now become noticeable, which makes itself felt in such a way that, with increasing centrifugal force, the oscillation modes of the unlike shaped bars continue to become alike and approach the oscillation line of the flexurally weak cable. The latter as well as lowest oscillation frequency of the cable are, however, unaffected by the distribution of mass over the length. (See Appendix.) Thus the marked discrepancies of C in the early experiments can scarcely be factual, as soon as the centrifugal exceed the elastic forces.

This was, in principle, the result of the writer's investigation (references 3, 4, and 6), made on the simple premises that the bending frequencies of two rotating bars, even if of marked difference in form, are practically alike so long as their static frequencies are the same. This also permits the inclusion of bar forms other than straight and untwisted, such as are found on propellers.

In these cases, rise of λ_0 due to twist, admittedly increases, but the additional frequency rise due to centrifugal force is determined largely by the mass distribution along the bar axis itself rather than the sectional orientation.

The centrifugal force effect is therefore seen to be practically unaffected by the propeller shape (that is, also from erroneous idealizations of the propeller shape). Once it had been determined, the problem resolved itself to defining the static frequency λ_o for each particular case. This is, of course, markedly dependent on the shape and other peculiarities of each propeller. Its true value is best obtained by test.

The mathematical exploration of these relationships (references 3 and 4) while, to be sure, no mathematically exact treatment, nevertheless afforded an amply safe approximate solution on the basis of Rayleigh's minimum equation:

$$\lambda^2 = [X_1 \lambda_o^2 + X_2 \omega^2]_{min} = F_1 \lambda_o^2 + F_2 \omega^2 \qquad (2)$$

wherein X_1, X_2 are functions of the bar form and the oscillation curve F_1, F_2 their values after formulating the minimum, with the characteristic of being practically invariant at constant λ_o against far-reaching changes in bar form.* Equation (2) was numerically computed and then replaced by the interpolation formula corrected for hub effect:

$$\frac{\lambda}{\lambda_o} = 1 + \frac{7\left(\frac{\omega}{\lambda_o}\right)^2}{6 + 7\left(\frac{\omega}{\lambda_o}\right)} \qquad (3)$$

so as to insure perfect freedom from tables or curves. (See fig. 8, reference 4.) But the correction factor for normal conditions is almost zero, as proved by Hansen and Mesmer's experimental data on section and inertia moment distribution (equation (32), reference 4), that it may be disregarded altogether. On many propellers the concept of rigid hub is definitely unjustified.**

*The modification of the bar form extended from cylindrical to linear and quadratically tapered bar together with taper of sectional inertia moment from linear to squared law.
**If, in special cases (and for $\omega > \lambda_o$) the omission of the propeller is inadvisable, a slight error in hub size means only a minor correction error.

Objections are raised in Hansen and Mesmer's report against the writer's effected idealization of blade form and the subsequent option in the method of calculating the determining root section. These objections are unfounded according to the above arguments advanced relative to (2), in a given case* in conjunction with the smallness of the hub effect, as soon as the true static frequency λ_o is known (by test, for example), as particularly premised in our last report (reference 6).

The use of expression (3) as an interpolation formula is wholly optional. It is not aptly chosen as may be seen from a quick comparison with results of the shape (1). It is more elucidative to expand the numerically obtained frequency λ^2 conformably in powers of ω^2 to conform with the manner of expressing equation (1). With four concrete values (which, in fact, is amply sufficient):

$$\left.\begin{array}{llll} \frac{\lambda}{\lambda_o} = 1 & \text{for} & \frac{\omega}{\lambda_o} = 0 \\ \quad = 1.16 & & \quad = 0.5 \\ \quad = 1.51 & & \quad = 1 \\ \quad = 1.95 & & \quad = 1.5 \end{array}\right\} \text{(reference 4, fig. 8)}$$

equation (3) is replaced by

$$\left(\frac{\lambda}{\lambda_o}\right)^2 = 1 + 1.43 \left(\frac{\omega}{\lambda_o}\right)^2 - 0.20 \left(\frac{\lambda}{\lambda_o}\right)^4 + 0.05 \left(\frac{\omega}{\lambda_o}\right)^6 \qquad (4)$$

applicable to $0 \leq \frac{\omega}{\lambda_o} \leq 1.5$. Then (4) affords the following: For $\frac{\omega}{\lambda_o} < 1$, the first and second terms suffice; for $\frac{\omega}{\lambda_o}$ approaching 1 and beyond, the higher powers are of importance. This decides, in addition, the coefficient of ω^2 in the proved manner, applicable to all practically possible propeller forms. (The value 1.43 itself cannot change even on completion of the series (4)).

With this manner of writing (4), the experimental proof of the theoretical frequency of the rotating propeller becomes quite apparent. To illustrate: Referring to Hansen and Mesmer's propeller test (reference 7), it first establishes one and the same dependence of the frequency on the r.p.m. for eight different propellers, and then this is repeated through formula (1) with C = 1.45. The

*That is, with extreme changes in blade form near the root from the elementary forms relative to which (2) proved "invariant."

measurements extend to resonance cycle $\omega = \lambda/2$ (two excitations per cycle). As far as this value, the agreement with (4) is practically complete (fig. 1). Extension to include the next resonance $\omega = \lambda/1.5$ (possible for propellers on 6- or 12-cylinder engines) should have been of interest. We think it would have shown discrepancies from the simple extrapolation with $C = 1.45$.

Our own experiments on elementary bars (reference 6) intended to confirm the theoretical premises by allowing for any possible blade characteristic (twist, camber, hub) as well as extension to higher ω/λ_0 values, had already proved the extended validity of the posed frequency formulas.*

Following these arguments on a proved formula of general validity, together with its experimental confirmation, should remove any doubt as to the definite determination of the fundamental mode of the rotating propeller. Incident to the determination of the static frequency λ_0 the following is noted:

Equation (4) is silent as regards the value of λ_0; its determination is a problem in itself which, however, is quickly and reliably obtained by experiment in nearly every case. Even the development of a new type generally affords an occasion for an experiment. If one is restricted to a mathematical treatment, there are certain graphical and mathematical methods available, such as those used by Southwell (reference 1), Hohenemser (reference 5), Hansen and Mesmer (reference 7), etc., whose results are, however, restricted for the reason that they are contingent upon certain omissions (twist, camber) and appraisals (metal edges, propeller surface, etc.) quite apart from the

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*Under these test conditions, any discrepancy between theory and test became, of necessity, quite apparent, as proved on two examples computed according to formula (1). Hansen and Mesmer's criticism (reference 7) is without basis when implying the insertion of $C = 1.52$ in (1) for comparison of the employed case of cylindrical bar. Berry (reference 11) as well as Southwell and Gough (reference 12), shows $C = 1.19$ for the cylindrical bar. Admittedly, the value 1.52 would "fit" better for the wedge, which is not at all surprising, according to (4), but just as accidental as that the $C = 1.08$, which fundamentally has the same claim to general validity, did not fit.

uncertain knowledge of material constants (Young's modulus, density) in many cases (wood screws).

Yet another frequently underestimated source of error is the presumption of absolutely rigid restraint at the root, because the frequency is quite responsive relative to the edge condition.* Several practical examples for computing the static frequency λ_o (for fundamental mode and first overtone) will be found in reference 7. The calculation was checked experimentally on 1:10 scale models. The agreement is fairly close although discrepancies up to 13 percent occur. One peculiar fact was that the calculated values are almost all above the experimental values, contrary to the expected opposite, because the bar twist is not allowed for. For this reason, it is advisable to make a check test.

In conclusion, we point to a recent report by Reissner (reference 8), who investigated the bending oscillations of propellers with reference to small camber and arbitrarily great initial twist. The latter exerts an effect of the first order on the flexural oscillations. The numerical data have not been published as yet, but are announced for a second report.

Overtones

The first publication on overtones of rotating bars is that by Hohenemser (reference 5). He obtained the overtone frequencies as fundamental frequencies of a bar modified in the nodal points by bearings, after the nodes had been established by means of a limiting condition.

In one example the centrifugal force factor is computed according to (1) for the first overtone at $C = 3.9$ and for the second, at $C = 12.2$. This was followed by Hansen and Mesmer's experiments (reference 7), which first revealed the occurrence of the first overtone along with the fundamental mode. The tests reveal $C = 4.4$ for the first overtone. Then Caldwell (reference 9) proposed $C = 3$ for the first, and $C = 4.5$ for the second overtone.

The given figures scatter considerably. But a close appraisal of the possible centrifugal force coefficients

*Concerning this condition, no certain conclusions are possible between model and full-scale test.

of the overtones is obtainable by fairly simple means, as shown by the writer's method utilized for computing the overtones (reference 6).

For a natural oscillation $y(\xi) \sin \lambda t$, the energy equation gives the true frequency at:

$$\lambda^2 = \frac{U_E(y)}{T(y)} + \frac{U_F(y)}{T(y)} = \lambda_o^{a\,2}(y) + \lambda_\omega^{a\,2}(y) \qquad (5)$$

when $y(\xi)$ is the true oscillation line. U_E is the potential energy of the elastic forces, U_F of the centrifugal force, and $T\lambda^2$ the kinetic energy; λ_o^a and λ_ω^a are the abbreviated summands shown at the left. Now, according to Rayleigh's law, the corresponding frequency computed from (5) varies only by a small amount of the second order - for both fundamental mode and overtones - for small variations of y. But, while this affords an upper limit, in the case of the fundamental mode, the approximately defined overtones do not of themselves reveal whether they represent too high or too low values.

Now the true natural function $y(\xi)$ in (5) must lie between the oscillation lines $y_E(\xi)$ and $y_F(\xi)$ which are, for the present valid, provided the variable centrifugal forces are considered effective on the bar. Since they are nearly alike in any case (as proved elsewhere), equation (5) affords approaches when either y_E or y_F is introduced. A third approach is obtained by writing y_E in the first summand of (5), and y_F in the second. Then each becomes equal to λ_o^2 and λ_ω^2 (λ_ω = frequency due to centrifugal force effect alone) and, since according to Rayleigh, the replacement of y for y_E or y_F, involves no appreciable error, we have

$$\lambda^2 = \lambda_o^2 + \lambda_\omega^2 \qquad (6)$$

as an approach. (However, we do not claim, as was possible with the fundamental frequency, that (6) implicitly denotes a lower limit.) The two first approaches are:

$$\lambda^2 = \lambda_o^{a\,2}(y_E) + \lambda_\omega^{a\,2}(y_E) = \lambda_o^2 + \lambda_\omega^{a\,2}(y_E) \qquad (6a)$$

and

$$\lambda^2 = \lambda_o^{a\,2}(y_F) + \lambda_\omega^{a\,2}(y_F) = \lambda_o^{a\,2}(y_F) + \lambda_\omega^2 \qquad (6b)$$

The form (6) is the most simple because normally the numerical determination of λ_0 may be foregone in favor of a test, thus leaving only λ_ω to be ascertained. The latter λ_ω, is the frequency with disregarded elastic forces; that is, simply the frequency of the flexural soft cable which merely depends on the mass distribution. It is readily computable and needs to be calculated no more than once because the mass distribution for practically all propeller forms can be quite exactly expressed as linear function of length, and any perceptible discrepancies from this assumption near the blade root may be neglected, since they are not likely to affect the cable frequency. The calculation of λ_ω up to the second overtone, is shown in the Appendix. Writing the obtained values in (6) gives:

$$\lambda^2 = \lambda_0 + 4.15\,\omega^2, \quad \text{1st overtone} \tag{7}$$

$$\lambda^2 = \lambda_0 + 9.2\,\omega^2, \quad \text{2d overtone} \tag{8}$$

as generally applicable approximation formulas.

On the other hand, the use of Hohenemser's formulas (reference 5) for the maximum-minimum properties of the higher natural frequencies, gives some consideration to the possible errors in the approximation formulas (6) to (6b). The assumption that the nodes of the bar subjected to elastic or centrifugal forces only, lie in both cases very close to each other, is itself legitimate. On the cylindrical bar, for instance, the node of the first overtone lies once at $\xi = 0.78$, and then at $\xi = 0.775$ $(= \sqrt{0.6}$ Appendix, equation (11)). Besides, as the frequency (on account of Hohenemser's stipulated limiting condition for the nodal points) is fairly indifferent to minor displacements of the nodal points from the true nodes, the overtones of the bar may be safely assumed as fundamental modes of a substitute bar with supports in the nodal points, which are available from the true oscillation line of the elastic bar or from the cable oscillation line. Then the frequency computed with y_E or y_F becomes the upper limit for the fundamental mode of the substitute bar and consequently also for the overtone of the original bar. However, it should be borne in mind that this analysis is not rigorous, and that in the unfavorable case where the discrepancy of the true from the assumed nodal positions reaches a "finite" value, the approximation frequency of the particular overtone can also become too low. Strictly speaking, Hohenemser's upper limits are not valid either, because they approach (6a).

With this reservation, however, all theorems applicable to the fundamental mode of the substitute bar supported in the nodal points, are equally valid for the overtone of the original bar. In other words, the values (6a) and (6b) are two upper limits and value (6) a lower limit for the true frequency.

Described preferably with the first overtone of the cylindrical bar for which all numerical values are exactly known, the following picture is obtained: With the conventional symbols for rigidity, mass, and length, the approach (6) gives in this example:

$$\lambda_o^2 + \lambda\omega^2 = 22.034^2 \frac{EJ}{m\, l^4} + 6\ \omega^2 < \lambda^2 \qquad \text{(I)}$$

(The value λ_o^2 is known, $\lambda\omega^2 = 6\ \omega^2$ is found in the Appendix.) Likewise, (6a) gives:

$$22.034^2 \frac{EJ}{m\, l^4} + 7.0\ \omega^2 = \lambda_o^2 + 1.167\ \lambda\omega^2 > \lambda^2 \qquad \text{(II)}$$

(equation (5), reference 5)*, and (6b) gives (y_F is equation (11) in the Appendix with $C = 6$):

$$22.913^2 \frac{EJ}{m\, l^4} + 6\ \omega^2 = 1.081\ \lambda_o^2 + \lambda\omega^2 > \lambda^2 \qquad \text{(III)}$$

and of the type shown in figure 2. (In reality, the curves are much closer together.) Figure 3 gives the percent discrepancy of frequency computed according to (I), (II), and (III). Now the true frequency is bounded by two upper limits, one of which is favorable at low, the other at higher cycles, and by a lower limit. It must lie within the hatched zone. Since the greatest possible movement within the zone of demarcation is less than 3 percent (at intersection of both upper limits: $\omega/\lambda_o = 0.28$), the accuracy of the calculations is good and the lower limit $\lambda_o^2 + \lambda\omega^2$ chosen as the most appropriate formula because of its simplicity and the unlimited validity range in ω. For the fundamental mode the conditions are not so favorable because y_E and y_F deviate more.

Following this discussion of the accuracy of approaches of form (6), we return to our practical problem, that is, the special formulas (7) and (8). The only doubtful

*Limited to the determination of λ_o - instead of 22.03 - 22.6 according to the employed iteration method.

factor lies in the general assumption of linear cross-section distribution. The first overtone - formula (7) - can be checked on the basis of Hansen and Mesmer's propeller tests, which gave $C = 4.4$. The measurements extend to $\lambda = 3\,\omega$. Figure 1 shows the extent of agreement between theory and test. Hohenemser's figure, $C = 3.9$, approaches that of equation (7) very closely. Caldwell's figure (fig. 1), $C = 3.0$, differs considerably and leads to objectionably great uncertainties, so that in view of the proven data, it may be ruled out.

The second flexural overtone of the propeller must also be included within the range of practical consideration, because in thin metal propellers, for instance, the sixth harmonic of the torque impulses may develop resonant oscillations at the r.p.m. occurring during its operation. In fact, there are cases in which the second overtone was claimed to be the cause of propeller damage (reference 10). Moreover, the second overtone is of interest because its frequency approaches the torsional frequency of the propeller (references 3 and 4), with a possibility of resonance.

Test data are available for the second overtone of the rotating propeller. Figure 1 shows the value for (8) as well as Hohenemser's $C = 12.2$ and Caldwell's $C = 4.5$. The first two values reveal tolerable discrepancies; but since Hohenemser's second overtone had been only roughly computed (reference 5), and our method had proved very satisfactory for the first overtone, the value of (8) may be considered as being the safer figure. Caldwell's $C = 4.5$ is evidently much too low.

After this discussion, the chapter on flexural propeller modes may, so far as concerns the determination of the oscillation frequency, be considered closed and its results as amply safe. (The principal formulas are collected in the Appendix, while figure 4 contains a practical example.)

The resonance r.p.m. for m excitations per rotation are given with

$$\frac{\omega}{\lambda_0} = \sqrt{\frac{1}{m^2 - C}} \qquad (9)$$

Resonance is possible only when $m^2 > C$; otherwise, the intersections of the straight line $\lambda = m\,\omega$ with the frequency curves are imaginary or infinite. If ω, as computed from (9), exceeds 1.5 times the static frequency for the fundamental mode, they are practically negligible because they lie above existing r.p.m.

APPENDIX

Cable Oscillations

With $y(x)$ = cable line (free cable end at $x = l$), t = time interval, m = mass, S = tension, the differential equation of the cable oscillation reads:

$$m \frac{\partial^2 y}{\partial t^2} = \frac{\partial}{\partial x}\left(S \frac{\partial y}{\partial x}\right), \quad \text{where} \quad S = \omega^2 \int_x^l m\, x\, dx$$

The new variable $z = 1 - \frac{x}{l}$ introduced for simplicity, gives:

1. For m = constant,

$$\frac{2}{\omega^2} \frac{\partial^2 y}{\partial t^2} = 2(1 - z) \frac{\partial y}{\partial z} + z(2 - z) \frac{\partial^2 y}{\partial z^2}$$

The equation of the fundamental modes

$$y = y(z) \sin \lambda t$$

in conjunction with the abbreviation $\frac{\lambda^2}{\omega^2} = C$ gives the customary differential equation of the second order:

$$\frac{d^2 y}{dz^2} z(2 - z) + \frac{dy}{dz} 2(1 - z) + 2\, C\, y = 0 \qquad (10)$$

of the type of Bessel's differential equation. Posing the solution as power series and defining its coefficient from the differential equation, results in:

$$y(z) = 1 - Cz + \frac{C(C-1)}{4} z^2 - \frac{C(C-1)(C-3)}{4 \times 9} z^3$$

$$+ \frac{C(C-1)(C-3)(C-6)}{4 \times 9 \times 16} z^4 \mp \qquad (11)$$

The limiting condition $y(1) = 0$ which, as is easily seen, can equally well be written in the form of infinite product:

$$y(1) = 0 = (C-1)\ (C-6)\ (C-15)\ (C-28)$$
$$[C-n\ (2n-1)]$$

with readily recognizable roots afford the frequencies $\left(C = \frac{\lambda^2}{\omega^2}\right)$: $\lambda^2 = \omega^2$, $\lambda^2 = 6\ \omega^2$, $\lambda^2 = 15\ \omega^2$ etc. (not 1, 3, 6, 10, etc.).

2. For $m = m_0\ z$, the differential equation

$$\frac{d^2y}{dz^2}\ z\ (3 - 2z) + \frac{dy}{dz}\ 6\ (1 - z) + 6\ C\ y = 0$$

replaces (10), resolving to:

$$y(z) = 1 - Cz + \frac{C(C-1)}{3}\ z^2$$

$$- \frac{C(C-1)\left(C - \frac{8}{3}\right)}{3 \times 6} z^3 + \frac{C(C-1)\left(C - \frac{8}{3}\right)(C-5)}{3 \times 6 \times 10} z^4$$

$$- \frac{C(C-1)\left(C - \frac{8}{3}\right)(C-5)\ (C-8)}{3 \times 6 \times 10 \times 15} z^5$$

$$+ \frac{C(C-1)\left(C - \frac{8}{3}\right)(C-5)\ (C-8)\left(C - \frac{35}{3}\right)}{3 \times 6 \times 10 \times 15 \times 21} z^6 \mp$$

The limiting condition $y(1) = 0$ affords (but not as readily as in the first case) the roots C and consequently, the frequencies: $\lambda^2 = \omega^2$, $\lambda^2 = 4.15\ \omega^2$, $\lambda^2 = 9.2\ \omega^2$ etc.

The oscillation line of the fundamental mode is a straight for any mass distribution, its frequency equal to the simple r.p.m.

Collection of Formulas

Discrepancies in the data and method of representation of different reports on flexural propeller oscillations suggested a survey and supplementary information for the purpose of coordination of the dependability of the mechanical principles for the calculation of the resonance r.p.m.

The fundamental frequency of any propeller is:

$$\left(\frac{\lambda}{\lambda_o}\right)^2 = 1 + 1.43 \left(\frac{\omega}{\lambda_o}\right)^2 - 0.20 \left(\frac{\omega}{\lambda_o}\right)^4 + 0.05 \left(\frac{\omega}{\lambda_o}\right)^6$$

valid for

$$0 \leq \frac{\omega}{\lambda_o} \leq 1.5$$

(A corresponding series expansion is necessary for the $0 \leq \frac{\omega}{\lambda_o} \leq \infty$ range.)

According to tests (reference 7):

$$\left(\frac{\lambda}{\lambda_o}\right)^2 = 1 + 1.45 \left(\frac{\omega}{\lambda_o}\right)^2$$

Test range $0 \leq \frac{\omega}{\lambda_o} \leq \frac{1}{2} \frac{\lambda}{\lambda_o}$, i.e., $0 \leq \frac{\omega}{\lambda_o} \leq 0.626$.

Theoretically, the first overtone is:

$$\left(\frac{\lambda}{\lambda_o}\right)^2 = 1 + 4.15 \left(\frac{\omega}{\lambda_o}\right)^2$$

valid for

$$0 \leq \frac{\omega}{\lambda_o} \leq \infty$$

Tests (reference 7) revealed:

$$\left(\frac{\lambda}{\lambda_o}\right)^2 = 1 + 4.4 \left(\frac{\omega}{\lambda_o}\right)^2$$

Test range $0 \leq \frac{\lambda}{\lambda_o} \leq \frac{1}{3} \frac{\lambda}{\lambda_o}$, i.e., $0 \leq \frac{\omega}{\lambda_o} \leq 0.466$.

No experimental data are available on the second overtone. The theoretical value is figured at:

$$\left(\frac{\lambda}{\lambda_o}\right)^2 = 1 + 9.2 \left(\frac{\omega}{\lambda_o}\right)^2$$

valid for $$0 \leq \frac{\omega}{\lambda_o} \leq \infty$$

The resonance r.p.m. lies at

$$\frac{\omega}{\lambda_o} = \sqrt{\frac{1}{m^2 - C}},$$

wherein λ_o is the static frequency, C the coefficient of $\left(\frac{\omega}{\lambda_o}\right)^2$ in the above equations for the fundamental frequency, first and second overtones, and m, the excitation per cycle. At higher revolutions $(\omega/\lambda_o \geq 1)$, formula (4) with the higher powers can be used for the fundamental mode.

Translation by J. Vanier,
National Advisory Committee
for Aeronautics.

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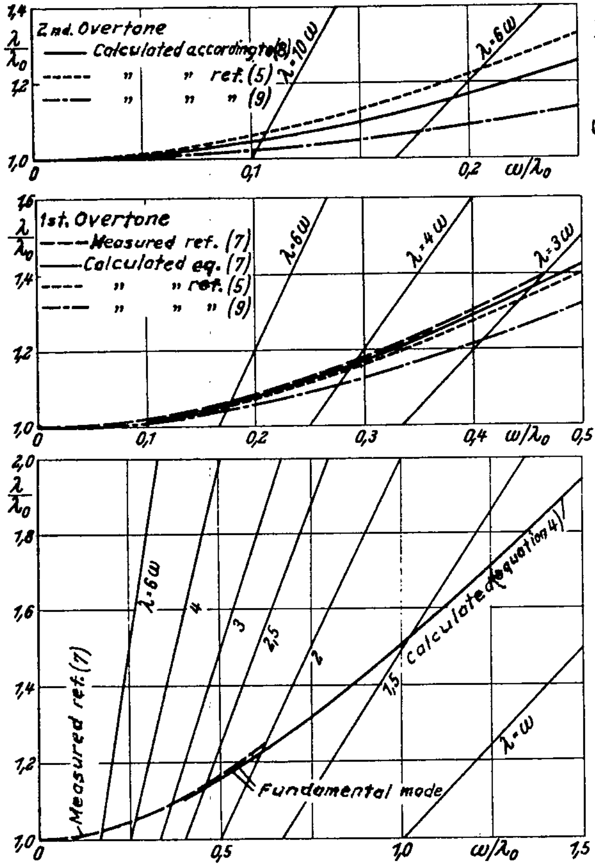

Figure 1.- Oscillation frequencies of rotating propeller versus revolutions. Frequency λ and revolution ω are referred to static frequency λ_0. The scales for ω/λ_0 have been chosen such that each plot approximately covers the normal range of propeller operation. The scale for λ/λ_0 is the same throughout so as to bring out the percentage frequency rise in the three cases.

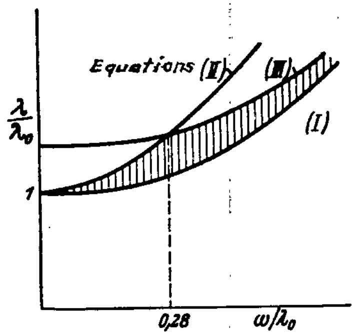

Figure 2.- Illustration of the three approaches I, II, and III.

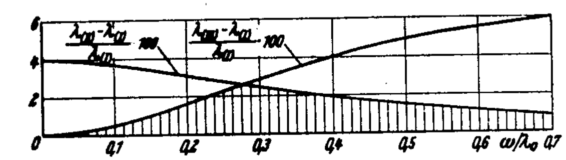

Figure 3.- Discrepancies of frequencies computed according to II, III in percent of frequency computed conformably to I.

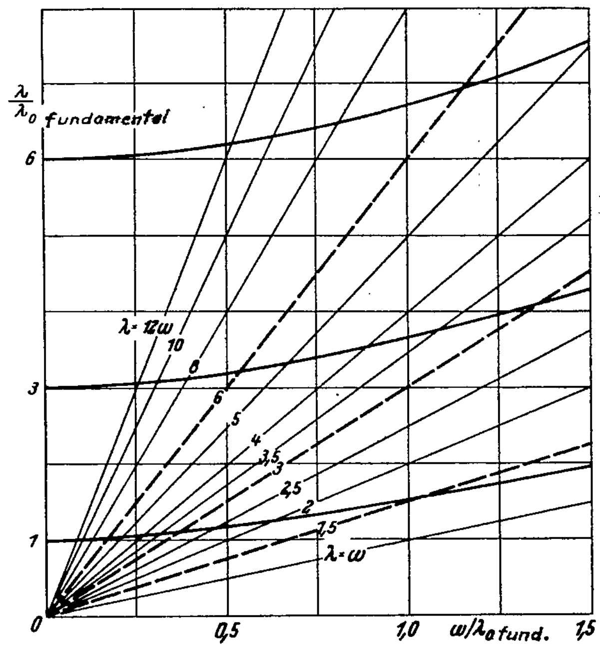

Figure 4.- Frequencies of fundamental mode and first and second overtone for a practically feasible example whereby the three static frequencies are as 1:3:6. The dashed lines intersect the frequency curves at the practically highest possible resonance cycles.